

CEADPAZ Problem set 1.2

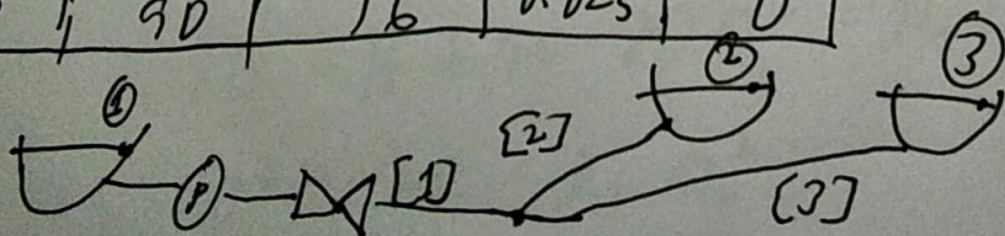
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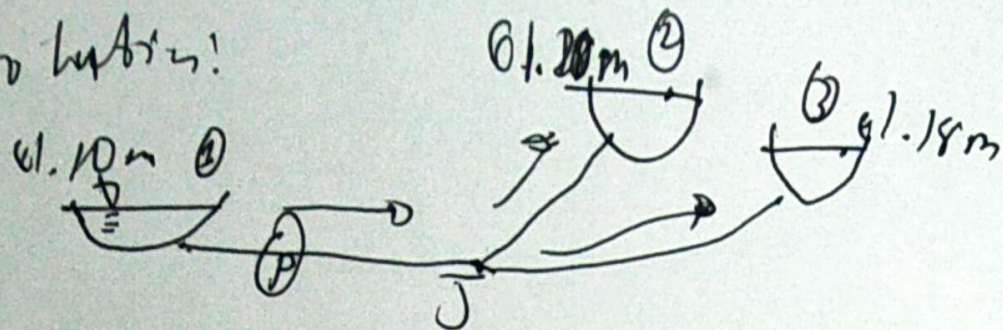
1.) (10 pts) Determine the water flow distribution in the simple pipe system shown in Figure 1 and the piezometric head at the junction using an ad hoc approach ($\Delta Q = Q_1 - Q_2 - Q_3$). Limit your iterations to 3. Assume constant friction factors.

The pump characteristic curve is $H_p = a - bQ^2$ where $a = 20m$, $b = 20 \frac{m^3}{s^2}$, $z_1 = 10m$, $z_2 = 20m$, $z_3 = 10m$
Pipe information!

Pipe	L (m)	D (cm)	f	K
1	30	24	0.020	2
2	60	20	0.015	0
3	90	16	0.025	0



So let's try:



$$H = \left(\frac{P}{\gamma} + z \right)_J$$

$$\left(\frac{P}{\gamma} + z \right)_1 + H + H_p = R_1 Q_1^2$$

$$H - \left(\frac{P}{\gamma} + z \right)_2 = R_2 Q_2^2$$

$$H - \left(\frac{P}{\gamma} + z \right)_3 = R_3 Q_3^2$$

$$Q_1 - Q_2 - Q_3 = 0$$

P is equal to zero since the reservoirs are exposed to air

$$10 - H + 20 - 20 Q_1^2 = R_1 Q_1^2$$

$$H - 20 = R_2 Q_2^2$$

$$H - 18 = R_3 Q_3^2$$

$$Q_1 - Q_2 - Q_3 = 0$$

$$L_0 = K \frac{D}{f}$$

$$L_{0,1} = 2 \cdot \frac{0.24}{0.020} = 28 \text{ m}$$

$$R_1 = \frac{8 P_1 [L_1 + L_{0,1}]}{8 \cdot 10^2 D_1^5} = 112.0698$$

$$R_2 = \frac{8 P_2 [L_2 + L_{0,2}]}{8 \cdot 10^2 D_2^5} = 232.3880$$

$$R_3 = \frac{8 P_3 [L_3 + L_{0,3}]}{8 \cdot 10^2 D_3^5} = 1772.9800$$

$$30 - H - 120 Q_1^2 = 112.0698 Q_1^2$$

$$\hookrightarrow 30 - H = 142.0698 Q_1^2$$

$$H - 20 = 232.3880 Q_2^2$$

$$H - 18 = 1772.9800 Q_3^2$$

$$Q_1 - Q_2 - Q_3 = 0$$

solving the system of equations to the interest of Q !

$$Q_1 = \sqrt{\frac{30-H}{142.0658}} = 0.0839 \sqrt{30-H}$$

$$Q_2 = \sqrt{\frac{H-20}{232.3850}} = 0.0656 \sqrt{H-20}$$

$$Q_3 = \sqrt{\frac{H-18}{1732.9800}} = 0.0237 \sqrt{H-18}$$

$$Q_1 - Q_2 - Q_3 = 0$$

$$0.0839 \sqrt{30-H} - 0.0656 \sqrt{H-20} - 0.0237 \sqrt{H-18} = 0 \quad f(H)$$

$$\frac{d}{dx} x^n = n x^{n-1}$$

$$-\frac{0.0420}{\sqrt{30-H}} - \frac{0.0328}{\sqrt{H-20}} - \frac{0.0115}{\sqrt{H-18}} = f'(H)$$

Newton - Raphson method?

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

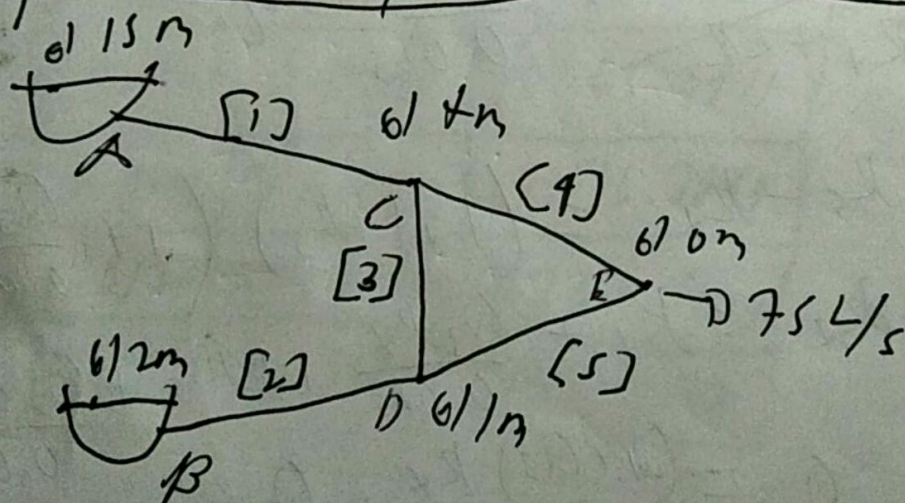
x_i	$f(x_i)$	$f'(x_i)$	x_{i+1}	$x_i - x_{i+1}$
25	-0.0218	-0.0375	24.4260	0.574035
24.4260	0.0000	0.5452	24.4260	-0.000008
24.4260	0.0000	0.5452	24.4260	-0.000009

$$H = 24.4260 \text{ m}$$

$$H = 24.43 \text{ m}$$

2) (10 pts) Determine the water flow dis-
tributions in the piping system shown in Figure 2
and the pressures at the intermediate nodes em-
ploying the Hardy Cross method of solution.
Limit your iterations to 3. (Assume $\beta = 2$)
Pipe information:

Pipe	L (m)	D (mm)	ϵ (mm)	ΣK
1	500	300	0.15	0
2	600	250	0.15	0
3	50	150	0.15	10
4	200	250	0.15	2
5	200	200	0.15	2



Solution:

$$75 \text{ } \gamma_s = 0.075 \text{ m}^3/\text{s}$$

$$f = 1.325 \left\{ \ln \left[0.27 \left(\frac{e}{D} \right) + 5.74 \left(\frac{1}{Re} \right)^{0.9} \right] \right\}^{-2}$$

assume $Re \approx \infty$!

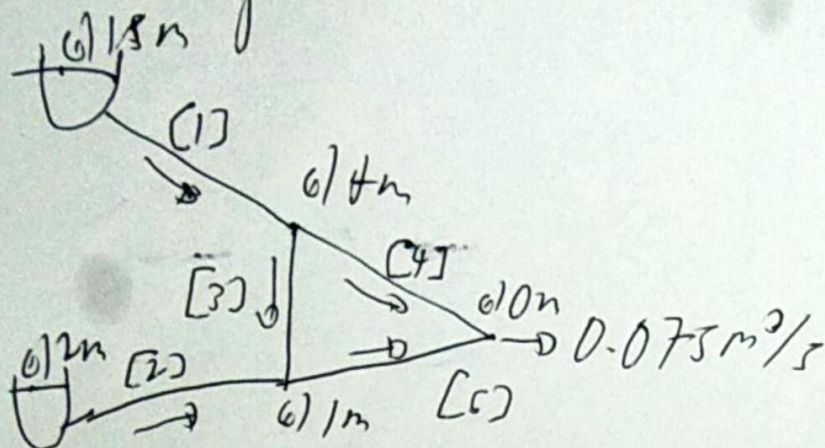
$$f = 1.325 \left\{ \ln \left[0.27 \left(\frac{e}{D} \right) \right] \right\}^{-2}$$

$$R = \frac{8(L + L_0)}{g \pi^2 D^5}$$

$$L_0 = K \frac{D}{f}$$

Pipes	f	L_0 (m)	R
1	0.0163	0.0000	12601.4109
2	0.0174	0.0000	50765.5411
3	0.0196	76.4314	137571.3754
4	0.0174	28.7458	15354.1634
5	0.0167	25.5512	8023.0077

Now Diagram



Find initial guess at:

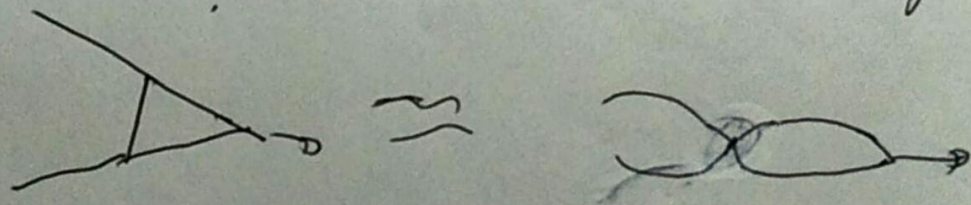
note! larger D , smaller Q

$$Q_4 + Q_5 = 0.075 \text{ m}^3/\text{s}$$

since for pipes in parallel, head losses are the same for each pipe!

$$R_4 Q_4^2 = R_5 Q_5^2$$

We could temporarily make the following assumption:



five notations:

$$Q_{45} = Q_4 + Q_5$$

$$R_{45} = \frac{1}{\frac{1}{R_4} + \frac{1}{R_5}}$$

$$W_{45} = R_{45} Q_{45}^2$$

$$R = \frac{[L^5]}{[T^2]}$$

$$Q_4^* = \sqrt{\frac{W_{45}}{R_{45}}}$$

$$Q_5^* = \sqrt{\frac{W_{45}}{R_5}}$$

$$R_{45} = 5671.8279$$

$$W_{45} = 5671.8279 (0.075)^2 = 31.9040$$

$$Q_4^* = 0.0406 \frac{m^3}{s}$$

$$Q_5^* = 0.0631 \frac{m^3}{s}$$

$$Q_4^* + Q_5^* = 0.1037 \frac{m^3}{s}$$

$$Q_4 = Q_4^* \left(\frac{0.075}{0.1037} \right) = 0.0294 \frac{m^3}{s}$$

$$Q_5 = Q_5^* \left(\frac{0.075}{0.1037} \right) = 0.0456 \frac{m^3}{s}$$

$$Q_4 + Q_5 = 0.075 \frac{m^3}{s}$$

$$Q_1 = Q_4 + Q_3$$

$$Q_2 + Q_3 = Q_5$$

$$Q_3 = Q_5 - Q_2$$

$$Q_1 = Q_4 + Q_5 - Q_2$$

$$Q_1 + Q_2 = Q_4 + Q_5$$

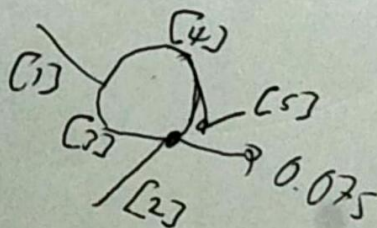
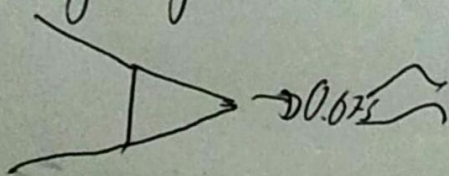
$$Q_1 + Q_2 = 0.075$$

$$Q_2 = 0.075 - Q_1$$

$$Q_3 = Q_5 - 0.075 + Q_1$$

$$Q_3 = -0.0254 + Q_1$$

using approximation!



$$Q_{43} = Q_4 + Q_3 = Q_1$$

$$R_{43} = 16967.2238$$

$$Q_4 = \sqrt{\frac{R_{43} Q_1^2}{R_4}} \Rightarrow \sqrt{\frac{R_4 Q_4^2}{R_{43}}} = Q_1 \therefore Q_1 =$$

$$Q_4^* = \sqrt{\frac{R_{43} Q_1^2}{R_4}}$$

$$Q_3^* = \sqrt{\frac{R_{43} Q_1^2}{R_3}}$$

$$Q_4^* + Q_3^* = Q_1^*$$

$$Q_1 \left(\sqrt{\frac{R_{43}}{R_4}} + \sqrt{\frac{R_{43}}{R_3}} \right) = Q_1^*$$

$$Q_1 (1.2875) = Q_1^*$$

$$\frac{Q_1}{Q_1^*} = \frac{1}{1.2875}$$

$$Q_4 = Q_4^* \left(\frac{1}{1.2875} \right)$$

$$Q_4 = \frac{1}{1.2875} \sqrt{\frac{R_{43} Q_1^2}{R_4}} \Rightarrow Q_4^2 R_4 (1.2875)^2 = R_{43} Q_1^2$$

$$Q_1 = \sqrt{\frac{Q_4^2 R_4 (1.2875)^2}{R_{43}}} = 0.0404 \frac{\text{m}^3}{\text{s}}$$

$$Q_3 = -0.0294 + Q_1 = 0.0110 \frac{\text{m}^3}{\text{s}}$$

$$Q_4 + Q_3 = 0.0404 \frac{\text{m}^3}{\text{s}}$$

$$Q_2 = 0.075 - Q_1 = 0.0346 \frac{\text{m}^3}{\text{s}}$$

initial guesses?

$$Q_1 = 0.0404 \frac{\text{m}^3}{\text{s}}$$

$$Q_2 = 0.0346 \frac{\text{m}^3}{\text{s}}$$

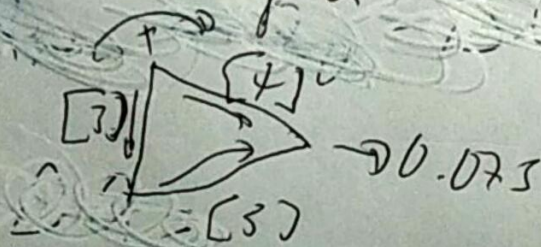
$$Q_3 = 0.0110 \frac{\text{m}^3}{\text{s}}$$

$$Q_4 = 0.0254 \frac{\text{m}^3}{\text{s}}$$

$$Q_5 = 0.0456 \frac{\text{m}^3}{\text{s}}$$

Handy cross iteration!

assign direction of Q :



Q_3 , negative

Q_4 , positive

Q_5 , negative

$$Q_{\text{new}} = Q_0 - \Delta Q$$

$$\Delta Q = \frac{\sum \sigma_i R_i |Q_i|^{n-1}}{\sum 2 R_i |Q_i|^{n-2}}$$

where σ_i is positive for clockwise flow, negative for counter-clockwise flow

First iteration

Name	Assumed Q	R	RQ/Q	2R/Q	New Q
Q ₃	-0.0110	137576.3454	-16.6467	42373.5144	-0.0108
Q ₄	0.0254	15354.1634	16.7250	15932.3473	0.0256
Q ₅	-0.0936	8023.0077	-16.6827	10243.7762	-0.0454
Sum	-0.0772		-16.6005	68541.6375	-0.0265

Second iteration

Name	Assumed Q	R	RQ/Q	2R/Q	New Q
Q ₃	-0.0108	137576.7454	-15.5218	41440.6518	-0.0105
Q ₄	0.0256	15354.1634	17.0057	16063.5815	0.0255
Q ₅	-0.0454	8023.0077	-16.5066	10189.7749	-0.0451
Sum	-0.0265		-15.4221	67633.6084	-0.0257

Third iteration

Name	Assumed Q	R	RQ/Q	2R/Q	New Q
Q ₃	-0.0105	137576.7454	-15.2771	40507.7853	-0.0103
Q ₄	0.0255	15354.1634	17.2847	16154.8165	0.0301
Q ₅	-0.0451	8023.0077	-16.3302	10139.9732	-0.0445
Sum	-0.0257		-14.2586	66837.5785	-0.0250

from iteration!

$$Q_3 = 0.0103 \frac{m^3}{s}$$

$$Q_4 = 0.0301 \frac{m^3}{s}$$

$$Q_5 = 0.0445 \frac{m^3}{s}$$

back substitutions of flow!

$$Q_1 = Q_3 + Q_4 = 0.0404 \frac{m^3}{s}$$

$$Q_2 + Q_3 = Q_5$$

$$Q_2 = Q_5 - Q_3 = 0.0346 \frac{m^3}{s}$$

$$Q_1 + Q_2 = 0.075 \frac{m^3}{s}$$

solving for pressure heads

$$15m - H_L = R_1 Q_1^2 \Rightarrow 15 - R_1 Q_1^2 = H_L$$

$$H_L = -12.7450m$$

$$\frac{P_L}{\gamma} + z_L = H_L \therefore P_L = -167307.65 Pa$$

$$15 - R_1 Q_1^2 = 0$$

$$Q_1 = \sqrt{\frac{15}{R_1}} = \frac{0.0297 \text{ m}^3/\text{s}}{0.0009 \text{ s}} \quad \left. \begin{array}{l} \text{find by or A} \\ \text{what } Q_1 \text{ would} \\ \text{be if } H_c = 0 \end{array} \right\}$$

$$15 - H_c = R_1 Q_1^2$$

$$\sqrt{\frac{15 - H_c}{R_1}} = Q_1$$

$$2m - H_D = R_2 Q_2^2$$

$$H_D = 2R_2 Q_2^2 - 2$$

$$H_D = -58.7750 \text{ m}$$

$$\frac{P_D}{\gamma} + z_D = H_D$$

$$P_D = -506352.75 \text{ Pa}$$

$$H_C - H_E = R_4 Q_4^2$$

$$H_C - R_4 Q_4^2 = H_E$$

$$H_E = -30.2841 \text{ m}$$

$$\left(\frac{P_E}{\gamma} + z_E \right) = H_E$$

$$P_E = -257087.621 \text{ Pa}$$

Find answers!

$$Q_1 = 4.04 \times 10^{-2} \frac{m^3}{s}$$

$$Q_2 = 3.46 \times 10^{-2} \frac{m^3}{s}$$

$$Q_3 = 1.02 \times 10^{-2} \frac{m^3}{s}$$

$$Q_4 = 3.01 \times 10^{-2} \frac{m^3}{s}$$

$$Q_5 = 4.45 \times 10^{-2} \frac{m^3}{s}$$

$$P_C = -164807.65 P_A$$

$$P_D = -586512.75 P_A$$

$$P_E = -297087.02 P_A$$